

Crossover between special and ordinary transitions in random semi-infinite Ising-like systemsZ. Usatenko^{1,*} and Chin-Kun Hu^{2,†}¹*Institute for Condensed Matter Physics, National Academy of Sciences of Ukraine, 1 Svientsitskii Street, UA-79011 Lviv, Ukraine*²*Institute of Physics, Academia Sinica, Taipei 11529, Taiwan*

(Received 16 June 2003; published 24 December 2003)

We consider the crossover behavior between special and ordinary surface transitions in three-dimensional semi-infinite Ising-like systems with random quenched bulk disorder. We calculate the surface crossover critical exponent Φ , the critical exponents of the layer α_1 , and local specific heats α_{11} by applying the field theoretic approach directly in three spatial dimensions ($d=3$) up to the two-loop approximation. The numerical estimates of the resulting two-loop series expansions for the surface critical exponents are computed by means of Padé and Padé-Borel resummation techniques. We find that Φ , α_1 , α_{11} obtained in the present paper are different from their counterparts of pure Ising systems. The obtained results support the idea that in a system with random quenched bulk disorder the plane boundary is characterized by a new set of critical exponents.

DOI: 10.1103/PhysRevE.68.066115

PACS number(s): 64.60.Fr, 05.70.Jk, 68.35.Rh, 75.40.Cx

I. INTRODUCTION

In recent decades the remarkable progress in understanding the critical behavior of real physical systems has been achieved from the application of the powerful field theoretical methods and renormalization group (RG) approach to the analysis of these systems. They can be used to perform with higher accuracy the numerical analysis of critical exponents and universal amplitude combinations for bulk phase transitions. There are a lot of publications related to this topic. The first few of them are [1–5]. For brevity we do not present all of them here. One can find a remarkable historical survey in [6]. The general review on the critical behavior of infinite randomly dilute spin models can be found in [7].

Moreover, these methods have given the possibility to investigate the preasymptotic behavior of infinite systems [8–10]. A series of field-theoretical methods developed and tested in the studies of bulk phase transitions have been extended to study the critical behavior of systems with boundaries. General reviews on surface critical phenomena are given in Refs. [11–13].

The presence of surfaces, which are inevitable in real systems, leads to additional complications. A typical model to study the critical phenomena in real physical systems restricted by a single planar surface is the semi-infinite model [11]. As it is known from the field-theoretic analysis of the continuum ϕ^4 model [12], we can take the influence of the surface into account by a quadratic surface term with coefficient c_0 , which describes the enhancement of the interactions at the surface, and additional surface fields h_1 . There are different surface universality classes, defining the critical behavior in the vicinity of boundaries, at temperatures close to the bulk critical point [$\tau=(T-T_c)/T_c \rightarrow 0$]. Each bulk universality class, in general, divides into several distinct surface universality classes. Three surface universality classes, called ordinary ($c_0 \rightarrow \infty$), special ($c_0 = c_{sp}^*$), and extraordinary ($c_0 \rightarrow -\infty$), are relevant for our case [12–14].

They meet at a multicritical point $(m_0^2, c_0) = (m_{0c}^2, c_{sp}^*)$, which corresponds to the special transition and is called the special point [15].

Most theoretical studies usually concentrate their attention on the investigation of critical behavior distinctly at the fixed points $c_0 = \pm\infty$ and $c_0 = c_{sp}^*$, respectively. At the present time, the theories of critical behavior of individual surface universality classes for pure isotropic systems [13,14,16–19] and systems with quenched surface-enhancement disorder [20–22] have been well developed. General irrelevance-relevance criteria of the Harris type for the systems with quenched short-range correlated surface-bond disorder were predicted in [20] and confirmed by Monte Carlo calculations [21,23]. The investigation of the surface critical behavior of the semi-infinite systems with random quenched bulk disorder at the ordinary and the special transitions have been studied by us [24,25]. The obtained results [24,25] have shown that such systems are characterized by the new set of surface critical exponents in comparison with the case of pure systems.

It is well known that experimental systems are typically characterized by the parameters different from values at the fixed point. However, our understanding of the situation in the crossover regions between different transitions is less complete. Investigations of the crossover behavior between different surface universality classes for pure isotropic systems have been published in a series of papers [18,19,26–29]. However, at the present time the question about the picture in the crossover regions between the different transitions for the semi-infinite systems with random quenched bulk disorder is still open. In the present paper we restrict our attention to the simplest case with surface field $h_1 = 0$ and investigate crossover behavior between special and ordinary transitions for semi-infinite Ising-like systems with random quenched bulk disorder. Here it should be mentioned that from the whole class of $O(N)$ symmetric N -vector models in d dimensions only the Ising model is the one of primary interest, because it satisfies the Harris criterion for the specific heat exponent $\alpha(d) \geq 0$ [30].

The proposed calculations are very important because they allow to understand the phenomenon of adsorption of

*Electronic address: pylp@ph.icmp.lviv.ua

†Electronic address: huck@phys.sinica.edu.tw

fluid mixtures in contact with a wall, as well as the critical behavior of so-called dilute magnets with the surfaces, which can be prepared by mixing an (anti)-ferromagnetic material with a nonmagnetic one.

The calculations are performed by applying a field theoretic approach directly in $d=3$ dimensions up to the two-loop order approximation. The numerical estimates of the resulting two-loop series expansions for the surface crossover exponent Φ from the special to the ordinary transition and surface critical exponents of the layer α_1 and local specific heats α_{11} are computed by means of the Padé [32] and Padé-Borel [33] resummation techniques. We find that Φ , α_1 , α_{11} obtained in the present paper are different from their counterparts of pure Ising systems.

II. MODEL

The Hamiltonian of the semi-infinite model under consideration with random quenched bulk disorder is given by

$$H = -\frac{1}{2} \sum_{\langle i,j \rangle \in \text{bulk}} J_{ij} p_i p_j s_i s_j - \sum_{\langle i,j \rangle \in \text{surface}} J'_{ij} s_i s_j, \quad (2.1)$$

where s_i and s_j are classical m -component spins located at the lattice sites i and j ; a nearest-neighbor bond $\langle i,j \rangle$ is said to belong to the surface region if both $i \in \text{surface}$ and $j \in \text{surface}$, in other cases they belong to the bulk region. The bulk interaction potential J_{ij} has the parallel to the plane translational invariance in the underlying lattice. The surface interaction potential J'_{ij} will never be invariant with respect to lattice translations parallel to the plane or perpendicular to it. The random site variable p_i and p_j have the probability distribution

$$P(p_i) = p \delta(p_i - 1) + p' \delta(p_i),$$

where $p' = 1 - p$ is the concentration of nonmagnetic impurities. As it is known, there are two possible ways to analyze the above random model. The first way is connected with direct averaging over random disorder using the method introduced by Lubensky [34]. The second possibility is to perform the configurational averaging of the free energy using the replica trick $n \rightarrow 0$,

$$F = -T \lim_{n \rightarrow 0} \frac{1}{n} (\langle Z^n \rangle_{\text{conf}} - 1),$$

where Z is the partition function of a configuration given by the Boltzman weight e^{-H} , as it was first done in the renormalization group (RG) calculations by Grinstein and Luther [35]. Performing the calculation in the spirit of the method introduced by Grinstein and Luther it is possible to show that the random model (2.1) is thermodynamically equivalent to the n -vector cubic anisotropic model ($m=1$) with an effective Hamiltonian of the Landau-Ginzburg-Wilson (LGW) type in semi-infinite space at the replica limit $n \rightarrow 0$

$$H(\vec{\phi}) = \int_0^\infty dz \int d^{d-1}r \left[\frac{1}{2} |\nabla \vec{\phi}|^2 + \frac{1}{2} m_0^2 |\vec{\phi}|^2 + \frac{1}{4!} v_0 \sum_{i=1}^n \phi_i^4 + \frac{1}{4!} u_0 (|\vec{\phi}|^2)^2 \right] + \int d^{d-1}r \frac{1}{2} c_0 \vec{\phi}^2, \quad (2.2)$$

where $\vec{\phi}(x)$ is an n -vector field with the components $\phi_i(x)$, $i=1, \dots, n$. Here m_0^2 is the ‘‘bare mass’’ representing a linear measure of the temperature difference from the critical point value. The values u_0 and v_0 are the usual ‘‘bare’’ coupling constants $u_0 < 0$ and $v_0 > 0$. The constant c_0 is related to the surface enhancement, which measures the enhancement of the interactions at the surface. It should be mentioned that the d -dimensional spatial integration is extended over a half-space $\mathbb{R}_+^d \equiv \{\mathbf{x} = (\mathbf{r}, z) \in \mathbb{R}^d | \mathbf{r} \in \mathbb{R}^{d-1}, z \geq 0\}$ bounded by a plane free surface at $z=0$. The fields $\phi_i(\mathbf{r}, z)$ satisfy the Dirichlet boundary condition in the case of ordinary transition: $\phi_i(\mathbf{r}, z) = 0$ at $z=0$ and the Neumann boundary condition in the case of special transition: $\partial_z \phi_i(\mathbf{r}, z) = 0$ at $z=0$ [14,17]. The model defined in Eq. (2.2) is restricted to translations parallel to the bounding surface, $z=0$. Thus, only parallel Fourier transformations in $d-1$ dimensions take place. It should be mentioned that the LGW model works good for sufficiently low spin dilution $1-p$ as long as the system is not too close to the percolation limit.

In order to consider the critical behavior in the crossover region and to calculate the crossover exponent Φ we should consider correlation functions with insertions of the surface operator ϕ_s^2 ,

$$G^{(N,M;L_1)}(\{\mathbf{x}_i\}, \{\mathbf{r}_j\}, \{\mathbf{R}_l\}) = \left\langle \prod_{i=1}^N \phi(\mathbf{x}_i) \prod_{j=1}^M \phi_s(\mathbf{r}_j) \prod_{l=1}^{L_1} \frac{1}{2} \phi_s^2(\mathbf{R}_l) \right\rangle, \quad (2.3)$$

which involve N fields $\phi(\mathbf{x}_i)$ at distinct points \mathbf{x}_i ($1 \leq i \leq N$) off the surface, M fields $\phi(\mathbf{r}_j, z=0) \equiv \phi_s(\mathbf{r}_j)$ at distinct surface points with parallel coordinates \mathbf{r}_j ($1 \leq j \leq M$), and L_1 insertions of the surface operator $\frac{1}{2} \phi_s^2(\mathbf{R}_l)$ at \mathbf{R}_l with ($1 \leq l \leq L_1$).

The corresponding parallel Fourier transform of the full free propagator takes form

$$G(\mathbf{p}, z, z') = \frac{1}{2\kappa_0} \left[e^{-\kappa_0 |z-z'|} - \frac{c_0 - \kappa_0}{c_0 + \kappa_0} e^{-\kappa_0(z+z')} \right], \quad (2.4)$$

with the standard notation $\kappa_0 = \sqrt{p^2 + m_0^2}$. Here, \mathbf{p} is the value of parallel momentum associated with $d-1$ translationally invariant directions in the system.

III. RENORMALIZATION

The formulation of the renormalization process for the random systems introduced by Grinstein and Luther [35] is

essentially the same as in the “pure” case [12,19]. From the other side, as it is known from the theory of semi-infinite systems [12,14,16,19], the bulk field $\phi(\mathbf{x})$ and the surface field $\phi_s(\mathbf{r})$ should be reparametrized by different uv-finite renormalization factors [12,19] $Z_\phi(u,v)$ and $Z_1(u,v)$. Thus we have $\phi = Z_\phi^{1/2} \phi_R$ and $\phi_s = Z_\phi^{1/2} Z_1^{1/2} \phi_{s,R}$. Besides, introducing the additional surface operator insertions $(1/2)\vec{\phi}_s^2(\mathbf{R}_l)$ requires additional specific renormalization factor $Z_{\phi_s^2}$

$$\phi_s^2 = [Z_{\phi_s^2}]^{-1} \phi_{s,R}^2.$$

The corresponding renormalized correlation functions involving N bulk, M surface fields, and L_1 surface operators $(1/2)\vec{\phi}_s^2(\mathbf{R}_l)$ can be written as

$$\begin{aligned} G_R^{(N,M,L_1)}(\mathbf{p}; m, u, v, c) &= Z_\phi^{-(N+M)/2} Z_1^{-M/2} Z_{\phi_s^2}^{L_1} \\ &\times G^{(N,M,L_1)}(\mathbf{p}; m_0, u_0, v_0, c_0). \end{aligned} \quad (3.1)$$

In the present paper we concentrate our attention on the correlation function $G^{(0,2,1)}(\mathbf{p}; m, u, v, c)$ involving two surface fields and a single surface operator insertion $\vec{\phi}_s^2(\mathbf{R}_l)$.

It is well known [19] that the uv singularities of the correlation function $G^{(N,M,L_1)}$ can be adsorbed through a mass shift $m_0^2 = m^2 + \delta m^2$ and surface-enhancement shift $c_0 = c + \delta c$. The renormalizations of the mass m , coupling constant u, v , and the renormalization factor Z_ϕ are defined by standard normalization conditions of the infinite-volume theory [7,35–38]. In order to adsorb uv singularities located in the vicinity of the surface, a surface-enhancement shift δc is required. In this connection the new normalization condition should be introduced (see Appendix A). Taking into account the normalization condition (A3) and expression for renormalized correlation function (3.1) it is possible to define the renormalization factor Z_{ϕ^2} in the form

$$[Z_{\phi_s^2}]^{-1} = Z_\parallel \left. \frac{\partial [G^{(0,2)}(0; m_0, u_0, v_0, c_0)]^{-1}}{\partial c_0} \right|_{c_0 = c_0(c, m, u, v)}. \quad (3.2)$$

It should be mentioned that the renormalization factor $Z_\parallel = Z_1 Z_\phi$ is defined via the standard normalization condition (A2) (see [19,25])

$$Z_\parallel^{-1} = 2m \left. \frac{\partial [G^{(0,2)}(p)]^{-1}}{\partial p^2} \right|_{p^2=0} = \lim_{p \rightarrow 0} \frac{m}{p} \frac{\partial [G^{(0,2)}(p)]^{-1}}{\partial p}. \quad (3.3)$$

Equation (3.2) enables us to considerably simplify the calculation of the correlation function $G^{(0,2,1)}$ with a surface operator $\vec{\phi}_s^2(\mathbf{R}_l)$ insertion.

It should be noted that all Z factors in the $d < 4$ case have finite limits at $\Lambda \rightarrow \infty$ (where Λ is a large-momentum cutoff). All factors mentioned above depend on the dimensionless variables u and v . Besides, the surface renormalization factors Z_1 and $Z_{\phi_s^2}$ depend on both u, v and the dimensionless ratio c/m . The last dependence on the ratio c/m plays the

crucial role in the investigation of the crossover behavior from the special transition ($c/m \rightarrow 0$) to the ordinary transition ($c/m \rightarrow \infty$).

IV. EXPANSION OF THE CORRELATION FUNCTION NEAR THE MULTICRITICAL POINT

As was indicated before, the main goal of the present work is to investigate the scaling critical behavior between special and ordinary transition and to calculate the crossover exponent Φ . In this connection let us consider the small deviations $\Delta c_0 = c_0 - c_{sp}^*$ from the multicritical point. The power expansion of the bare correlation functions $G^{(N,M)}(\mathbf{p}; m_0, u_0, v_0, c_0)$ in terms of this small deviation Δc_0 has a form

$$\begin{aligned} G^{(N,M)}(\mathbf{p}; m_0, u_0, v_0, c_0) &= \sum_{L_1=0}^{\infty} \frac{(\Delta c_0)^{L_1}}{L_1!} G^{(N,M,L_1)}(\mathbf{p}; m_0, u_0, v_0, c_{sp}^*). \end{aligned} \quad (4.1)$$

Based on Eq. (3.1), we rewrite the right-hand part of Eq. (4.1) in terms of the renormalized correlation functions and renormalized variable $\Delta c = [Z_{\phi_s^2}(u, v)]^{-1} \Delta c_0$ and obtain

$$\begin{aligned} Z_\phi^{-(N+M)/2} (Z_1)^{-M/2} G^{(N,M)}(\mathbf{p}; m_0, u_0, v_0, c_0) &= \sum_{L_1=0}^{\infty} \frac{(\Delta c)^{L_1}}{L_1!} G_R^{(N,M,L_1)}(\mathbf{p}; m, u, v). \end{aligned} \quad (4.2)$$

The preceding equation in straightforward fashion defines the correspondent renormalized correlation functions defined in the vicinity of the multicritical point

$$\begin{aligned} G_R^{(N,M)}(\mathbf{p}; m, u, v, \Delta c) &= Z_\phi^{-(N+M)/2} (Z_1)^{-M/2} G^{(N,M)}(\mathbf{p}; m_0, u_0, v_0, c_0). \end{aligned} \quad (4.3)$$

It is easy to see that these correlation functions depend on the dimensionless variable $\bar{c} = \Delta c/m$. Thus, the correlation functions $G_R^{(N,M)}(\mathbf{p}; m, u, v, \Delta c)$ satisfy correspondent Callan-Symanzik equations [19,40]

$$\begin{aligned} \left[m \frac{\partial}{\partial m} + \beta_u(u, v) \frac{\partial}{\partial u} + \beta_v(u, v) \frac{\partial}{\partial v} + \frac{N+M}{2} \eta_\phi(u, v) \right. \\ \left. + \frac{M}{2} \eta_1^{sp}(u, v) - [1 + \eta_{\bar{c}}(u, v)] \bar{c} \frac{\partial}{\partial \bar{c}} \right] \\ \times G_R^{(N,M)}(\mathbf{p}; m, u, v, \Delta c) = \Delta G_R, \end{aligned} \quad (4.4)$$

where the inhomogeneous part ΔG_R should be negligible in the critical region similarly as that takes place in the case of infinite field theory. The resulting homogeneous equation differs from the standard bulk Callan-Symanzik (CS) equation [41–43] in that it involves the additional surface-related function η_1^{sp} and term $-[1 + \eta_{\bar{c}}(u, v)] \bar{c} (\partial/\partial \bar{c})$, where

$$\begin{aligned} \eta_1^{sp}(u,v) &= m \left. \frac{\partial}{\partial m} \right|_{\text{FP}} \ln Z_1(u,v) \\ &= \beta_u(u,v) \frac{\partial \ln Z_1(u,v)}{\partial u} + \beta_v(u,v) \frac{\partial \ln Z_1(u,v)}{\partial v} \Big|_{\text{FP}} \end{aligned} \quad (4.5)$$

and

$$\begin{aligned} \eta_c(u,v) &= m \left. \frac{\partial}{\partial m} \right|_{\text{FP}} \ln Z_{\phi_s^2}(u,v) \\ &= \beta_u(u,v) \frac{\partial \ln Z_{\phi_s^2}(u,v)}{\partial u} \\ &\quad + \beta_v(u,v) \frac{\partial \ln Z_{\phi_s^2}(u,v)}{\partial v} \Big|_{\text{FP}}. \end{aligned} \quad (4.6)$$

It should be mentioned that functions $\beta_u(u,v)$, $\beta_v(u,v)$ and $\eta_\phi(u,v)$ appearing in Eq. (4.4) are the usual bulk RG functions. The symbol ‘‘FP’’ indicates that the above value should be calculated at the infrared-stable random fixed point (FP) of the underlying bulk theory.

V. SCALING CRITICAL BEHAVIOR AT THE MULTICRITICAL POINT

The asymptotic scaling critical behavior of the correlation functions can be obtained through a detailed analysis of the CS equations of Eq. (4.4), as was proposed in [41,44] and employed in the case of the semi-infinite systems in [19,28,29]. Our present investigations of the scaling critical behavior are in complete analogy with the scheme mentioned above [19,44] (see Appendix B). Taking into account the scaling form of the renormalization factor $Z_{\phi_s^2}$ of Eq. (B1) and the relation $m \sim \tau^\nu$, we obtain for Δc and for the scaling variable \bar{c} the next asymptotic dependences

$$\Delta c \sim m^{-\eta_c(u^*,v^*)} \Delta c_0, \quad \Delta c \sim \tau^{-\nu \eta_c(u^*,v^*)} \Delta c_0 \quad (5.1)$$

and

$$\bar{c} \sim m^{-[1 + \eta_c(u^*,v^*)]} \Delta c_0, \quad \bar{c} \sim \tau^{-\Phi} \Delta c_0, \quad (5.2)$$

where

$$\Phi = \nu [1 + \eta_c(u^*,v^*)] \quad (5.3)$$

is the surface crossover critical exponent. Equation (5.2) explains the physical meaning of the surface crossover exponent as a value which characterizes the measure of deviation from the multicritical point. The second equations in Eqs. (5.1) and (5.2) indicate about nonanalytic temperature dependence of the renormalized surface-enhancement deviation Δc . Taking into account the above-mentioned results from the CS equation we obtain the next asymptotic scaling form of the surface correlation function $G^{(0,2)}$

$$\begin{aligned} G^{(0,2)}(p; m_0, u_0, v_0, c_0) \\ \sim m^{-\gamma_{11}^{sp/\nu}} G_R^{(0,2)}\left(\frac{p}{m}; 1, u^*, v^*, m^{-\Phi/\nu} \Delta c_0\right) \\ \sim \tau^{-\gamma_{11}^{sp}} G(pt^{-\nu}; 1, \tau^{-\Phi} \Delta c_0), \end{aligned} \quad (5.4)$$

where $\gamma_{11}^{sp} = \nu(1 - \eta_{\parallel})$ is the local surface susceptibility exponent and $\eta_{\parallel}^{sp} = \eta_1^{sp} + \eta$ is the surface correlation exponent [45]. It is easy to see that the asymptotic scaling critical behavior of the surface correlation function for the systems with random quenched bulk disorder is characterized by the new crossover exponent $\Phi(u^*, v^*)$, which belongs to the universality class of the random model. In the next section, we will calculate the surface crossover exponent Φ of the semi-infinite systems with random quenched bulk disorder.

VI. THE PERTURBATION SERIES UP TO TWO LOOPS

According to Eqs. (5.3) and (4.6) the calculation of the crossover critical exponent Φ is connected with the calculation of the renormalization factor $Z_{\phi_s^2}$ via Eq. (3.2). The usual bulk uv singularities which are present in correlation function $[G^{(0,2)}(0)]^{-1}$ can be removed by the method similar to those reported in Refs. [19], [24], [37], [39] with the help of the standard mass renormalization procedure.

The second step of our calculation is to remove the uv divergences which are connected with the presence of the surface in the system. The surface uv singularities of the inverse surface correlation function $[G^{(0,2)}(0)]^{-1}$ can be removed by performing the surface enhancement renormalization which is defined by Eq. (A1.1). For convenience we can rewrite the normalization condition of Eq. (A1) in the form

$$Z_{\parallel} [G^{(0,2)}(0; m_0, u_0, v_0, c_0)]^{-1} = m + c \quad (6.1)$$

for the inverse unrenormalized surface correlation function $[G^{(0,2)}(0)]^{-1}$. Performing the differentiation of the above-mentioned normalization condition with respect to $\partial/\partial c_0$ and taking into account Eq. (3.2) we obtain for the renormalization factor Z_{ϕ^2} the next equation

$$Z_{\phi_s^2} = \frac{\partial c_0}{\partial c}, \quad (6.2)$$

where $c_0 = c + \delta c$ and

$$\delta c = (Z_{\parallel}^{-1} - 1)(m + c) + \sigma_0(0; m, c_0 = c + \delta c). \quad (6.3)$$

Here $\sigma_0(0; m, c_0)$ denotes the sum of loop diagrams of all orders in $[G^{(0,2)}(0; m, u_0, v_0, c_0)]^{-1}$ (see [19,25]). Among them σ_1 corresponds to the one-loop graph, σ_2 denotes the melonlike two-loop diagrams

$$\sigma_2 = \dots \left(\frac{\mathbf{G}}{\mathbf{G}} \right) \dots - \frac{1}{2\kappa} \left(\frac{\bar{}}{\bar{}} \right) + \frac{m^2}{2\kappa} \frac{\partial}{\partial k^2} \left(\frac{\bar{}}{\bar{}} \right) \mathbf{K} \Big|_{k^2=0}, \quad (6.4)$$

σ_3 and σ_4 represent the reducible and irreducible two-loop diagrams in $[G^{(0,2)}(0; m, u_0, v_0, c_0)]^{-1}$, respectively. Here the full lines with labels “G” denote the full free propagator of Eq. (2.4). Equation (6.2) can be resolved by using the method of sequential iteration. As a result of the first order of the perturbation theory (one-loop approximation) at general spatial dimensions $d < 4$, we obtain

$$Z_{\phi_s^2}^{(1)} = 1 - \frac{T_1}{2} \frac{\pi^{-(d-1)/2}}{16} \frac{\Gamma(\epsilon)}{\Gamma\left(\frac{\epsilon+1}{2}\right)} \times \left[1 - 2^{(1+\epsilon)/2} {}_2F_1\left(\frac{3-\epsilon}{2}; \frac{1+\epsilon}{2}; \frac{3+\epsilon}{2}; \frac{1}{2}\right) \right], \quad (6.5)$$

where ${}_2F_1(\dots)$ is the hypergeometric function and coefficient $T_1 = [(n+2)/3]\bar{u}_0 + \bar{v}_0$ includes dimensionless effective expansion parameters $\bar{u}_0 = u_0 m^{-\epsilon}$ and $\bar{v}_0 = v_0 m^{-\epsilon}$ which are identified through the standard vertex renormalization. In the case of massive theory the value of ϵ is not necessarily restricted to be small and the above expressions hold for any relevant dimensions $d < 4$. The presence of the Euler gamma function $\Gamma(\epsilon)$ indicates the existence of dimensional poles $1/\epsilon$ when $\epsilon \rightarrow 0$.

According to Eqs. (3.2) and (6.5) for $\eta_{\bar{c}}$ at a one-loop order in the case of general dimensions up to $d=4$ we obtain

$$\eta_{\bar{c}} = \frac{T_1}{2} \frac{\epsilon}{1+\epsilon} \pi^{-d/2} 2^{-d} \Gamma(\epsilon/2) \times \left[1 - 2^{(1+\epsilon)/2} {}_2F_1\left(\frac{3-\epsilon}{2}; \frac{1+\epsilon}{2}; \frac{3+\epsilon}{2}; \frac{1}{2}\right) \right]. \quad (6.6)$$

At the random fixed point of order $O(\sqrt{\epsilon})$ [46] ($K_4 u^* = -3\sqrt{6\epsilon/53}$, $K_4 v^* = 4\sqrt{6\epsilon/53}$, where the geometric normalization factor $K_4 = 1/(8\pi^2)$), Eq. (6.6) in the limit $\epsilon \rightarrow 0$ leads to

$$\lim_{\epsilon \rightarrow 0} \eta_{\parallel}^{sp} = \lim_{\epsilon \rightarrow 0} \eta_{\bar{c}} = -\sqrt{\frac{6\epsilon}{53}}. \quad (6.7)$$

This result coincides with that obtained by Ohno and Okabe [46]. At $\epsilon \rightarrow 0$ for the surface crossover exponent Φ in one-loop calculations we obtain

$$\Phi = \frac{1}{2} - \frac{1}{4} \sqrt{\frac{6\epsilon}{53}}. \quad (6.8)$$

In the case of three spatial dimensions ($d=3$) the renormalization factors Z_1 and $Z_{\phi_s^2}$ are finite and their one-loop expressions do not coincide. At one-loop order, we obtain

$$\eta_{\bar{c}} \approx -0.596 \quad \text{and} \quad \Phi = 0.286. \quad (6.9)$$

In the next order of the perturbation theory we restrict our attention only to the case of $d=3$ dimensions. Thus after the

surface-enhancement renormalization and performing the Feynman integrals in analogy with [19,25] and carrying out the vertex renormalizations of bare dimensionless parameters $\bar{u}_0 = u_0/8\pi m$ and $\bar{v}_0 = v_0/8\pi m$

$$\bar{u}_0 = \bar{u} \left(1 + \frac{n+8}{6} \bar{u} + \bar{v} \right), \quad \bar{v}_0 = \bar{v} \left(1 + \frac{3}{2} \bar{v} + 2\bar{u} \right), \quad (6.10)$$

we obtain a second-order series expansion for the renormalization factor $Z_{\phi_s^2}$ in terms of new renormalized coupling constants \bar{u} and \bar{v} ,

$$Z_{\phi_s^2}(\bar{u}, \bar{v}) = 1 + \frac{n+2}{3} \left(\ln 2 - \frac{1}{4} \right) \bar{u} + \left(\ln 2 - \frac{1}{4} \right) \bar{v} + \frac{n+2}{3} C(n) \bar{u}^2 + 2C(n) \bar{u} \bar{v} + C(1) \bar{v}^2, \quad (6.11)$$

where $C(n)$ is a function of the replica number n , defined by

$$C(n) = A - B - \frac{n}{2} \ln 2 + \frac{n+2}{2} \ln^2 2 + \frac{2n+1}{12}, \quad (6.12)$$

and $A = 0.202428$, $B = 0.678061$ are integrals originating from the two-loop melon-like diagrams. Combining the renormalization factor $Z_{\phi_s^2}$ with the one-loop pieces of the β functions $\beta_{\bar{u}}(\bar{u}, \bar{v}) = -\bar{u}(1 - [(n+8)/6]\bar{u} - \bar{v})$ and $\beta_{\bar{v}}(\bar{u}, \bar{v}) = -\bar{v}(1 - (3/2)\bar{v} - 2\bar{u})$ according to Eq. (4.6), we obtain the desired series expansion for $\eta_{\bar{c}}$,

$$\eta_{\bar{c}}(u, v) = -2 \frac{n+2}{n+8} \left(\ln 2 - \frac{1}{4} \right) u - \frac{2}{3} \left(\ln 2 - \frac{1}{4} \right) v - 8 \left[3 \frac{n+2}{(n+8)^2} D(n) u^2 + \frac{2D(n)}{n+8} uv + \frac{D(1)}{9} v^2 \right], \quad (6.13)$$

where

$$D(n) = A - B + \frac{n+2}{3} \ln^2 2 - \frac{n+1}{2} \ln 2 + \frac{17n+22}{96}, \quad (6.14)$$

and renormalized coupling constants u and v , normalized in a standard fashion $u = [(n+8)/6]\bar{u}$ and $v = (3/2)\bar{v}$. In common, Eq. (6.13) gives a result for the model with the effective Hamiltonian of the Landau-Ginzburg-Wilson type with cubic anisotropy in the semi-infinite space (2.2) with a general number n of order parameter components. Our calculations are connected with the investigation of the critical behavior of *semi-infinite random Ising-like* systems by taking the replica limit $n \rightarrow 0$. Hence, we obtain

TABLE I. Surface critical exponents involving the RG function $\eta_{\bar{c}}$ at the Jug fixed point $u^* = -0.60509$, $v^* = 2.39631$ (two-loop order).

Exp.	$\frac{O_1}{O_2}$	$\frac{O_{1i}}{O_{2i}}$	[0/0]	[1/0]	[0/1]	[2/0]	[0/2]	[11/1]	[1/11]	[R]	R_i^{-1}	$f(\alpha_1, \nu, \eta)$
$\eta_{\bar{c}}$	-0.8	-1.5	0.00	-0.574	-0.365	0.150	-0.152	-0.281	-0.268	-0.313	-0.280	-0.164
α_1	-1.7	-7.6	0.50	0.051	0.190	0.312	0.220	0.201	0.213	0.185	0.211	0.211
α_{11}	-1.1	-2.8	0.00	-0.574	-0.365	-0.036	-0.268	-0.324	-0.306	-0.351	-0.313	-0.222
Φ	-0.5	-0.5	0.50	0.375	0.389	0.652	0.658	0.451	0.452	0.444	0.445	0.567

$$\eta_{\bar{c}} = -\frac{1}{2} \left(\ln 2 - \frac{1}{4} \right) u - \frac{2}{3} \left(\ln 2 - \frac{1}{4} \right) v - 8 \left[\frac{3}{32} D(0) u^2 + \frac{D(0)}{4} uv + \frac{D(1)}{9} v^2 \right]. \quad (6.15)$$

The knowledge of $\eta_{\bar{c}}$ gives access to the calculation of the crossover critical exponents Φ via the scaling relation of Eq. (5.3). Besides, we can calculate the critical exponents α_1 and α_{11} of the layer and local specific heats via the usual scaling relations [12]

$$\alpha_1 = \alpha + \nu - 1 + \Phi = 1 - \nu(d - 2 - \eta_{\bar{c}}),$$

$$\alpha_{11} = \alpha + \nu - 2 + 2\Phi = -\nu(d - 3 - 2\eta_{\bar{c}}). \quad (6.16)$$

The above critical exponents should be calculated at the standard infrared-stable random fixed (FP) point of the underlying bulk theory [47] $u^* = -0.60509$ and $v^* = 2.39631$, as it is usually accepted in the massive field theory.

VII. NUMERICAL RESULTS

For each of the surface critical exponents mentioned above and the crossover exponent Φ we obtain from Eq. (6.15) at $d = 3$ a double series expansion in powers of u and v truncated at the second order [48]. In order to perform the analysis of these perturbative series expansions and to obtain accurate estimates of the surface critical exponents a powerful resummation procedure must be used. One of the simplest ways is to perform the double Padé analysis [32]. This should work well when the series behaves in lowest orders “in a convergent fashion.” Another way is to perform the double Padé-Borel analysis [33] for these series. The usage of the Padé-Borel resummation procedures are possible in the case when the terms in the series are alternating in sign

[4]. The results of our calculations at Jug random fixed point [47] are represented in Table I. The quantities O_1/O_2 and O_{1i}/O_{2i} represent the ratios of magnitudes of first-order and second-order perturbative corrections appearing in direct and inverse series expansions. The larger (absolute) value of these ratios indicate about the better apparent convergence of truncated series [19].

The values $[p/q]$ (where $p, q = 0, 1$) in Table I are simply Padé approximants which represent the partial sums of the direct and inverse series expansions up to the first and second order. The nearly diagonal two-variable rational approximants of the types $[11/1]$ and $[1/11]$ give at $u = 0$ or $v = 0$ the usual $[1/1]$ Padé approximant [32]. The results of the Padé-Borel analysis of the direct R and the inverse R^{-1} series expansions give numerical estimates of the surface critical exponents with a high degree of reliability in the frames of the present approximation scheme. One can see from Table I that the largest (absolute) value of the quantities O_1/O_2 and O_{1i}/O_{2i} exists for the inverse series expansion of the surface critical exponent α_1 . It indicates that from the inverse series expansion for the surface critical exponent α_1 , which represents the best convergence properties, we obtain the most reliable estimate. Substituting this value $\alpha_1 = 0.211$ together with the standard bulk value $\nu = 0.678$ into the scaling relations (5.3) and (6.16), we have obtained the remaining critical exponents that are present in the last column of Table I. The deviations of these estimates from the other estimates of the table give a rough measure of the achieved numerical accuracy.

The results of the similar analysis of the perturbative series expansions of the surface critical exponent at random fixed point $u^* = -0.6524$, $v^* = 2.4203$ [49] are presented in Table II for comparison. In a similar way the most reliable estimate is obtained for an inverse series expansion of α_1 . The results of substituting $\alpha_1 = 0.208$ and $\nu = 0.679$ into scaling laws (5.3) and (6.16) are presented in the last column of Table II. As is easy to see from a comparison of the results of Table I and Table II, the difference in the ways of the β

TABLE II. Surface critical exponents involving the RG function $\eta_{\bar{c}}$ at the fixed point $u^* = -0.6524$, $v^* = 2.4203$ (two-loop order).

Exp.	$\frac{O_1}{O_2}$	$\frac{O_{1i}}{O_{2i}}$	[0/0]	[1/0]	[0/1]	[2/0]	[0/2]	[11/1]	[1/11]	[R]	R_i^{-1}	$f(\alpha_1, \nu, \eta)$
$\eta_{\bar{c}}$	-0.82	-1.55	0.0	-0.570	-0.363	0.124	-0.168	-0.287	-0.272	-0.318	-0.283	-0.166
α_1	-1.82	-9.56	0.5	0.054	0.191	0.300	0.215	0.197	0.209	0.182	0.208	0.208
α_{11}	-1.12	-3.08	0.0	-0.570	-0.363	-0.060	-0.278	-0.331	-0.311	-0.357	-0.317	-0.226
Φ	-0.47	-0.50	0.5	0.376	0.389	0.641	0.643	0.449	0.450	0.442	0.444	0.566

functions resummation do not have essential influence on the values of the surface critical exponents. The difference between the final results of Table I and Table II are 1.2% for η_c , 1.4% for α_1 , 1.8% for α_{11} , and 0.2% for Φ .

For evaluation of the reliability of the results obtained in the two-loop approximation, we have performed an additional calculation of the surface critical exponents from $\alpha_1 = 0.211$ and six-loop perturbation theory results [7] for a bulk critical exponent of the correlation length $\nu = 0.678(10)$. We have obtained $\eta_c = -0.164$, $\alpha_{11} = -0.222$ and for the surface crossover critical exponent $\Phi = 0.567$. This indicates the good stability of our results obtained in the frames of the two-loop approximation scheme.

VIII. SUMMARY

We have studied the crossover critical behavior between special and ordinary surface transitions of three-dimensional quenched random semi-infinite Ising-like systems. We find that the asymptotic scaling critical behavior of the surface correlation function for the systems with random quenched bulk disorder is characterized by the crossover exponent $\Phi(u^*, v^*)$, which belongs to the universality class of the random model. We have calculated the crossover critical exponent Φ and critical exponents of the layer α_1 and local specific heats α_{11} by applying the field theoretic approach directly in three dimensions up to the two-loop approximation. We have performed a rational double Padé and double Padé-Borel analysis of the resulting perturbation series expansions for the surface critical exponents in order to find the best numerical values. The final numerical values of the surface critical exponents α_1 , α_{11} and crossover exponent Φ for the systems with quenched random bulk disorder in the frames of the present approximation scheme are

$$\alpha_1 = 0.211, \quad \alpha_{11} = -0.222, \quad \Phi = 0.567. \quad (8.1)$$

These values evidently different from their counterparts of pure Ising systems [19,39]

$$\alpha_1 = 0.279, \quad \alpha_{11} = -0.182, \quad \Phi = 0.539. \quad (8.2)$$

We suggest that the obtained results could stimulate further experimental and numerical investigations of the surface critical behavior of random systems.

ACKNOWLEDGMENTS

We would like to thank Dr. M. Shpot for a fruitful discussion and C.-K.H. would like to thank the National Science Council of Republic of China (Taiwan) for supporting this work under Grant No. NSC 91-2112-M-001-056.

APPENDIX A

In order to specify δc , Z_1 , and Z_{ϕ^2} , we require that [19,39]

$$G_R^{(0,2)}(\mathbf{p}; m, u, v, c) \Big|_{\mathbf{p}=\mathbf{0}} = \frac{1}{m+c}, \quad (A1)$$

$$\frac{\partial G_R^{(0,2)}(\mathbf{p}; m, u, v, c)}{\partial p^2} \Big|_{\mathbf{p}=\mathbf{0}} = -\frac{1}{2m(m+c)^2}, \quad (A2)$$

and the correspondent normalization condition for the correlation function $G^{(0,2,1)}$ with the insertion of the surface operator $(1/2)\phi_s^2$,

$$G_R^{(0,2,1)}(\mathbf{p}; m, u, v, c) \Big|_{\mathbf{p}=\mathbf{0}} = \frac{1}{(m+c)^2}. \quad (A3)$$

Equation (A3) is motivated by the fact that the bare correlation function $G^{(0,2,1)}(0; m_0, u_0, v_0, c_0)$ may be written as derivative $-(\partial/\partial c_0)G^{(0,2)}(0; m_0, u_0, v_0, c_0)$. This equation simplifies considerably the calculation of the correlation function with insertions of surface operator $(1/2)\phi_s^2$.

From Eq. (A1) it is easy to see that the special point is located at $m=c=0$, because at this point the divergence of the bulk and the surface correlation length and susceptibility is observed. At $c=0$ the surface normalization conditions are simplified and yield $c_0 = c_{sp}^*$. This point corresponds to the multicritical point (m_{0c}^2, c_{sp}^*) at which special transition takes place. On the other hand, the above-mentioned equation implies also that the surface correlation length and the susceptibility are finite at the ordinary transition, because in this case we have $c > 0$ when $m \rightarrow 0$. This latter case corresponds to the situation when the surface remains “noncritical” at the bulk transition temperature.

APPENDIX B

As it is usually accepted in the massive field theory, the variable m is identified with the inverse bulk correlation length ξ^{-1} and is proportional to τ^ν , where $\tau = (T - T_c)/T_c$. Following the scheme proposed in [44], we can perform the integration of Eq. (4.5), Eq. (4.6), and expressions for the RG functions $\beta_u(u, v)$, $\beta_v(u, v)$, and $\eta_\phi(u, v)$. This gives the following asymptotic dependencies at $m \rightarrow 0$:

$$|u - u^*| \sim m^{\omega_u} \quad \text{where } \omega_u = \beta'_u(u^*, v^*),$$

$$|v - v^*| \sim m^{\omega_v} \quad \text{where } \omega_v = \beta'_v(u^*, v^*),$$

$$Z_\phi \sim m^\eta \quad \text{where } \eta = \eta_\phi(u^*, v^*),$$

$$Z_1 \sim m^{\eta_1^{sp}(u^*, v^*)},$$

$$Z_{\phi_s^2} \sim m^{\eta_c(u^*, v^*)}. \quad (B1)$$

As follows from these expressions, the variables u and v deviate from their fixed values u^* and v^* by different scaling laws with various values of ω_u and ω_v . The scaling laws [see Eq. (B1)] have the similar form as in the case of the pure systems [19], but renormalization factors are characterized by another value of the critical exponents which belong to the universality class of random model.

- [1] K. G. Wilson and J. Kogut, *Phys. Rep.* **12C**, 75 (1974).
- [2] *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic Press, London, 1976), Vol. 6.
- [3] K. G. Wilson and M. E. Fisher, *Phys. Rev. Lett.* **28**, 240 (1972).
- [4] G. A. Baker, Jr., B. G. Nickel, and D. I. Meiron, *Phys. Rev. B* **17**, 1365 (1978).
- [5] J. C. Le Guillou and J. Zinn-Justin, *Phys. Rev. B* **21**, 3976 (1980).
- [6] S. S. Schweber, *Phys. Today* **46**(11), 34 (1993).
- [7] A. Pelissetto and E. Vicari, *Phys. Rev. B* **62**, 6393 (2000).
- [8] C. Bagnuls and C. Bervillier, *Phys. Rev. B* **32**, 7209 (1985).
- [9] C. Bagnuls, C. Bervillier, D. I. Meiron, and B. G. Nickel, *Phys. Rev. B* **35**, 3585 (1987).
- [10] R. Schloms and V. Dohm, *Nucl. Phys. B* **328**, 639 (1989).
- [11] K. Binder, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic Press, London, 1983), Vol. 8, pp. 1–144.
- [12] H. W. Diehl, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic Press, London, 1986), Vol. 10, pp. 75–267.
- [13] H. W. Diehl, *Int. J. Mod. Phys. B* **11**, 3503 (1997); e-print cond-mat/9610143.
- [14] H. W. Diehl and S. Dietrich, *Z. Phys. B: Condens. Matter* **42**, 65 (1981).
- [15] The coupling m_0 is the “bare mass,” representing a linear measure of the temperature difference from the critical point value.
- [16] H. W. Diehl and S. Dietrich, *Z. Phys. B: Condens. Matter* **50**, 117 (1983).
- [17] H. W. Diehl, S. Dietrich, and E. Eisenriegler, *Phys. Rev. B* **27**, 2937 (1983).
- [18] H. W. Diehl and A. Ciach, *Phys. Rev. B* **44**, 6642 (1991).
- [19] H. W. Diehl and M. Shpot, *Nucl. Phys. B* **528**, 595 (1998).
- [20] H. W. Diehl and A. Nüsser, *Z. Phys. B: Condens. Matter* **79**, 69 (1990); **79**, 79 (1990).
- [21] M. Pleimling and W. Selke, *Eur. Phys. J. B* **1**, 385 (1998).
- [22] H. W. Diehl, *Eur. Phys. J. B* **1**, 401 (1998).
- [23] F. Iglói, P. Lajkó, W. Selke, and F. Szalma, *J. Phys. A* **31**, 2801 (1998).
- [24] Z. E. Usatenko, M. A. Shpot, and Chin-Kun Hu, *Phys. Rev. E* **63**, 056102 (2001).
- [25] Z. E. Usatenko and Chin-Kun Hu, *Phys. Rev. E* **65**, 066103 (2002).
- [26] E. Brezin and S. Leibler, *Phys. Rev. B* **27**, 594 (1983).
- [27] U. Ritschel and P. Czerner, *Phys. Rev. Lett.* **77**, 3645 (1996).
- [28] A. Ciach and U. Ritschel, *Nucl. Phys. B* **489**, 653 (1997).
- [29] A. Maciolek, A. Ciach, and A. Drzewinski, *Phys. Rev. E* **60**, 2887 (1999).
- [30] The Harris criterion [31] states that the presence of a noncorrelated (or short-range-correlated) quenched disorder in the bulk has a nontrivial effect on the critical behavior of magnetic systems only if the specific heat critical exponent α of a pure magnet is of positive value.
- [31] A. B. Harris, *J. Phys. C* **7**, 1671 (1974).
- [32] G. A. Baker, Jr., *Essentials of Padé Approximants* (Academic, New York, 1975).
- [33] G. A. Baker, Jr., B. G. Nickel, M. S. Green, and D. I. Meiron, *Phys. Rev. Lett.* **36**, 1351 (1976).
- [34] T. C. Lubensky, *Phys. Rev. B* **11**, 3573 (1975).
- [35] G. Grinstein and A. Luther, *Phys. Rev. B* **13**, 1329 (1976).
- [36] E. Brézin, J. C. Le Guillou, and J. Zinn-Justin, in *Phase Transition and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic Press, New York, 1976), Vol. 6.
- [37] G. Parisi, *J. Stat. Phys.* **23**, 49 (1980).
- [38] D. V. Pakhnin and A. I. Sokolov, *Phys. Rev. B* **61**, 15130 (2000).
- [39] H. W. Diehl and M. Shpot, *Phys. Rev. Lett.* **73**, 3431 (1994).
- [40] M. Shpot, *Condens. Matter Phys.* **10**, 143 (1997).
- [41] J. Zinn-Justin, *Euclidean Field Theory and Critical Phenomena* (Oxford University Press, New York, 1989).
- [42] G. Parisi, *Statistical Field Theory* (Addison-Wesley, Redwood City, CA, 1988).
- [43] C. Itzykson and J.-M. Drouffe, *Statistical Field Theory* (Cambridge University Press, Cambridge, 1989), Vol. I.
- [44] C. Bagnuls and C. Bervillier, *Phys. Rev. B* **24**, 1226 (1981).
- [45] The calculation of the surface correlation exponent η_{\parallel} and other surface critical exponents of the special transition was performed in [25].
- [46] K. Ohno and Y. Okabe, *Phys. Rev. B* **46**, 5917 (1992).
- [47] G. Jug, *Phys. Rev. B* **27**, 609 (1983).
- [48] The individual RG expansions for the surface critical exponent were obtained from the above scaling relations (5.3) and (6.16) (at $d=3$) and by combining Eq. (6.15) with the $n \rightarrow 0$ limit of the two-loop series expansion for ν
- $$\nu = \frac{1}{2} + \frac{(n+2)u}{4(n+8)} + \frac{v}{12} - \frac{1}{108} \left[\frac{(n+2)(38-27n)}{2(n+8)^2} u^2 + \frac{11}{54} v^2 + \frac{38-27n}{3(n+8)} uv \right].$$
- [49] N. A. Shpot, *Phys. Lett. A* **133**, 125 (1988); N. A. Shpot, Institute for Theoretical Physics Report No. ITF-88-16P, Kiev, 1988 (in Russian).